

Radiative electroweak symmetry breaking in the extra dimensions scenarios

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We study the radiative spontaneous electroweak symmetry breaking in the extra dimensions scenarios of the standard model extension proposed by Antoniadis *et al.*, Dienes *et al.* and Pomarol *et al.*. In the framework of multi-scale effective theory when viewing from the ultraviolet cutoff scale down to the low energy scale, we find that the effects of Kaluza-Klein excitations of bosons can change the sign of the Higgs mass term of the standard model from positive to negative and therefore trigger the electroweak symmetry breaking at 1.6 (2) TeV or so if the compactification scale is assumed to be 0.8 (1.5) TeV or so. New particle contents beyond the SM or supersymmetry are not necessary for this mechanism. We conclude that in the extra dimension scenarios, the radiative correction can naturally induce the desired electroweak symmetry breaking.

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The extra dimension (ED) scenario of the standard model (SM) extension is fascinating and is under the vigorous investigation (refer [1] for a brief review), since it provides a new paradigm which is within the reach of near future experiments and beyond technicolor and supersymmetry (SUSY). Many efforts have been invested in the ED model construction [2]. However, a realistic model requires to answer how to break the electroweak symmetry.

In the SM, to trigger the spontaneous electroweak symmetry breaking (SEWSB), the mass term of the Higgs potential is set to be negative by hand. Although the Higgs mechanism is successful in explaining the masses of bosons and fermions and their mixing, it is quite unnatural in this respect. The underlying reason for the SEWSB of the SM is still an open question [3].

There are two main models to explain the SEWSB of the SM: technicolor and SUSY models. In technicolor models, Higgs field is regarded as a composite field of other more fundamental fields. The SEWSB is due to the fermion condensate. The SEWSB of the technicolor version in the ED has been intensively investigated (refer [4] for a review).

In SUSY models, the radiative symmetry breaking mechanism [5] is well known [6]. Due to the largeness of top Yukawa couplings, the mass term of the Higgs doublet coupled to u-type quarks can be driven from positive to negative and the SEWSB can be triggered naturally. However, we would like to point out that beside the the largeness of top Yukawa coupling, another important factor which is always under-emphasized is the fact that in softly broken SUSY models there exist couplings with mass dimension (trilinear couplings, for instance) and masses of heavy superpartners (especially the scalars of top-quark partner which contribute to the radiative SEWSB constructively). Without these important terms, the radiative breaking mechanism of the SUSY can not work.

In some ED scenarios, there are a lot of heavy particle spectrum in the reduced 4D effective theory (For example, the Kaluza-Klein (KK) excitations of both vector and Higgs boson are possibly quite heavy). Is it possible for these heavy particles to induce the desired SEWSB at a few TeV, as the scalars of SUSY do? In this work, we will answer this question and study the radiative mechanism in non-supersymmetric ED extension of the SM proposed by Antoniadis *et al.* [7], Dienes *et al.* [8] and Pomarol *et al.* [9], where gauge and Higgs bosons are assumed to propagate in the bulk. The case where only vector bosons live in the bulk and the universal case where all fermions are also assumed to live in the bulk are discussed.

Firstly, we examine how it is possible to spontaneously break symmetry in a two-real-scalar system. And we use the multi-mass-scale effective potential method (MEPM) given in the references [10] and [11]. In a theory with more than one scale, MEPM can avoid large logarithms and preserves the validity of perturbation theory. The basic ingredients of the MEPM include the renormalization group equation (RGE) method and the decoupling theorem [13]. One of the advantages of the MEPM is that by using different effective field theories [12] which are dependent on the field scales, it is convenient to solve the RGEs of the effective potential. Because the RGEs are just the same ones in each interval between mass thresholds.

The effective Lagrangian of the system defined at the ultraviolet cutoff scale (UV) is assumed to be

$$L_{UV} = \frac{1}{2} \left[\sum_{i=1}^2 (\partial^\nu H_i)^\dagger \partial_\nu H_i - m_{H_i}^2 (\mu_{UV}) H_i^\dagger H_i \right] - V(H_1, H_2), \quad (1)$$

where $H_i, i = 1, 2$ are two real scalar fields. We assume $|m_{H_1}^2(\mu_{UV})| < m_{H_2}^2(\mu_{UV})$.

Motivated by the effective 4D Lagrangian reduced from

the 5D one, the potential $V(H_1, H_2)$ is simply assumed to have the form

$$V(H_1, H_2) = \frac{\lambda(\mu_{UV})}{4!}(H_1^4 + 6H_1^2 H_2^2 + H_2^4), \quad (2)$$

where $\lambda(\mu_{UV})$ is positive. The potential is invariant under the transformation $H_1 \rightarrow -H_1$ and $H_2 \rightarrow -H_2$. In the UV, We can choose $m_{H_i}^2(\mu_{UV})$, $i = 1, 2$, and $\lambda(\mu_{UV})$ as the three free parameters owned by the system. The symmetry is assumed to be unbroken and $m_{H_i}^2(\mu_{UV})$ are positive.

At the low energy scale (IR), after H_2 decouples at its threshold scale $m_{H_2}^*$, the obtained effective theory at low energy scale μ_{IR} can be written as

$$L_{eff} = \frac{1}{2}(\partial^\nu H_1)^\dagger \partial_\nu H_1 - m_{H_1}^2(\mu_{IR}) H_1^\dagger H_1 - V(H_1) + \dots, \quad (3)$$

where the dots represent the omitted irrelevant terms and $V(H_1)$ can be simply expressed as

$$V(H_1) = \frac{\lambda_1(\mu_{IR})}{4!} H_1^4. \quad (4)$$

This effective Lagrangian also owns a Z_2 reflection symmetry.

The one-loop RGEs of this system are listed below

$$\frac{dm_{H_1}^2(t)}{dt} = \frac{1}{4\pi}[\alpha_\lambda m_{H_1}^2(t) + \alpha_\lambda m_{H_2}^2(t)\theta(\mu - m_{H_2})], \quad (5)$$

$$\frac{dm_{H_2}^2(t)}{dt} = \frac{1}{4\pi}[\alpha_\lambda m_{H_1}^2(t) + \alpha_\lambda m_{H_2}^2(t)\theta(\mu - m_{H_2})], \quad (6)$$

$$\frac{d\alpha_\lambda}{dt} = \frac{1}{4\pi}[3\alpha_\lambda + 3\alpha_\lambda\theta(\mu - m_{H_2})], \quad (7)$$

where $t = \ln\mu/\mu_{IR}$, $\alpha_\lambda = \lambda/(4\pi)$, and $\theta(\mu - m_{H_2})$ is the Heaviside theta function.

At the IR, we assume that $m_{H_1}^2(\mu_{IR})$ is negative and the Z_2 symmetry is broken. The IR free parameters can be chosen as v (i.e. the vacuum expectation value (VEV) of the field H_1), $\alpha_\lambda(\mu_{IR})$, and m_{H_2} . The $\alpha_\lambda(\mu_{IR})$ should be positive as required by the stability of the vacuum.

The value of $m_{H_1}(\mu_{IR})$ is fixed by the relation $m_{H_1}^2(\mu_{IR}) = -2\pi/3\alpha_\lambda(\mu_{IR})v^2$. By solving the RGEs given in eqs. (5-7), it is obvious that below the threshold value of m_{H_2} , the self coupling λ will become stronger and stronger with the running of energy scale and the negative mass term $m_{H_1}^2$ will be simply driven to be further negative.

When the RGEs run across the threshold value of the m_{H_2} , the degree of freedom of the H_2 is activated. If the

condition $|m_{H_1}^2(t)| < |m_{H_2}^2(t)|$ is satisfied for $\mu \geq m_{H_2}$, it is sufficient for the existence of a scale μ_{cri} where the sign of $m_{H_1}^2$ can be flipped from negative to positive. At the scale μ_{cri} , the VEV of the H_1 become zero and the broken symmetry is restored. This condition can be solved out from eqs. (5-7) as

$$|\frac{2\pi}{3}\alpha_\lambda v^2| < m_{H_2}^2(1 - \frac{\alpha_\lambda}{4\pi} \ln \frac{m_{H_2}^2}{\mu_{IR}})^{\frac{1}{3}}. \quad (8)$$

Therefore, in the IR, for a fixed v , choosing a proper point in the parameters space of m_{H_2} and α_λ which satisfies the condition given in eqn. (8) as an IR input, and running the RGEs from the IR up to the UV, we can get a point of in the parameter space determined ($m_{H_i}^2(\mu_{UV})$, $i = 1, 2$ and $\alpha_\lambda(\mu_{UV})$) in the UV. With this set of parameters as an UV input, and running the RGEs from the UV down to the IR, we can definitely get the desired radiative spontaneous symmetry breaking at the critical scale μ_{cri} .

The lesson we learn from this case is that if an IR input of a system can trigger the symmetry restoring, then when running from up down to bottom, the UV input determined by the IR input can definitely trigger the symmetry breaking at the same critical scale, since the RGEs are differential equations and are solvable with a specified boundary condition.

Now let's examine the case in the ED scenarios. We consider a simple extension of the SM to 5D [9] (It is straightforward to generalize our discuss to cases with high number of extra dimensions). The fifth space-like dimension x_5 is assumed to compactify on the orbifold S^1/Z_2 . The 5D Lagrangian is defined as

$$\mathcal{L}_{5D} = -\frac{1}{4}F_{MN}^2 + |D_M H|^2 - V(H) + L_{GF} + \left[i\bar{\psi}_i \sigma^\mu D_\mu \psi_i + Y_F \bar{\psi}_L H \psi_R \right] \delta(x_5), \quad (9)$$

where $F_{MN} = \partial_M A_N - \partial_N A_M + g_5[A_M, A_N]$, which is the gauge field tensor defined in 5D, and $N, M = 0, 1, 2, 3, 5$. The group generators index is omitted. Gauge fields A_M and Higgs weak doublet field H have mass dimension $3/2$. g_5 and Y_F are the gauge coupling and the Yukawa coupling, respectively, and have mass dimension $-1/2$. \mathcal{L}_{GF} is the gauge fixed term. $V(H)$ is the usual Higgs potential and has the form

$$V(H) = \mu^2 H^\dagger H + \frac{\lambda}{4}(H^\dagger H)^2, \quad (10)$$

μ^2 and λ are the mass term and self coupling of H boson, respectively. And λ has mass dimension -1 . Here μ^2 is assumed to be positive and the $SU(2) \times U(1)$ symmetry is assumed to be preserve when the compactification occurs. According to the power counting law, the gauge theory defined in 5D is non-renormalizable, due to the fact that the couplings have negative mass dimension.

*Although H_2 should decouples at $M_{H_2} = m_{H_2} + \lambda H_1/2$ as stated in [11], due to the large mass of KK excitations, the difference between m_{H_2} and M_{H_2} is omitted here.

The fields living in the bulk can be defined to be even under the Z_2 -parity and can be Fourier-expanded as

$$A_M(H)(x_\mu, x_5) = \sum_{n=0}^{\infty} \cos \frac{nx_5}{R_c} A_{M(5D)}^n(H_{(5D)}^n)(x_\mu), \quad (11)$$

where R_c is the compactification size of the fifth dimension, and $A_{M(5D)}^{(n)}(H_{(5D)}^n), n \neq 0$ are KK excitations. Zero modes are localized on the 3-brane and are fields defined in the SM. Substituting eqn. (11) into eqn. (9), and rescaling fields and parameters with $\lambda_{5D} = 2\pi R \lambda_{4D}$, $g_{5D}(Y_{u5D}) = \sqrt{2\pi R} g_{4D}(Y_{u4D})$, $A_{\mu 5D}^0(H_{5D}^0) = A_{\mu 4D}^0(H_{4D}^0)/\sqrt{2\pi R}$, $A_{\mu 5D}^n(H_{5D}^n, A_{55D}^n) = A_{\mu 4D}^n(H_{4D}^n, A_{54D}^n)/\sqrt{\pi R}$, ($n \neq 0$). Then we get the effective Lagrangian in 4D defined at the UV Λ_{UV} which has a form

$$\mathcal{L}^{eff} = \mathcal{L}_{SM} + \delta\mathcal{L}^{ED}, \quad (12)$$

where \mathcal{L}_{SM} is just the Lagrangian of the SM in 4D, and $\delta\mathcal{L}^{ED}$ contains all interactions of KK excitations with zero modes on 3-brane. The effective theory owns a $SU_c(3) \times SU_L(2) \times U_Y(1)$ symmetry. In order to be compatible with the SM, A_5 (the fifth component of vector boson field) is assumed to have no zero mode. So by utilizing the standard dimension reduction procedure, we specify a basic effective Lagrangian of extra dimension scenarios with the symmetry of the SM.

Two features of the model are remarkable [8]. The first one is universal KK excitation spectrum of fields propagating in the bulk. The mass values of each level of KK excitations can be expressed as $m_i = i/R_c$, and are independent of other quantum numbers, spin and charge, for instance.

The second one is that the presence of infinite towers of KK states makes the effective theory non-renormalizable and this is an intrinsic feature for high dimensions theory. In order to have a renormalizable effective theory, an explicit ultraviolet cutoff Λ is introduced to truncate the infinite KK excitation.

The RGEs of gauge couplings take the form $d\alpha_g/dt = 2b_g\alpha_g/(2\pi)$, and up to one-loop level, b_g is defined as

$$b_g = -\frac{11}{3}C_g(G) + \frac{2}{3}\sum_f T_g(\Psi_f) + \frac{1}{3}T_g(H) \\ + [-\frac{11}{3}C_g(G) + \frac{1}{6}C_g(A_5) + \frac{1}{3}T_g(H)]N_{KK}, \quad (13)$$

where $\alpha_g = g^2/(4\pi)$, and $N_{KK} = \sum_{i=1} \theta(\mu - m_i)$, counting the number of activated KK excitations. To understand the RGEs of gauge couplings, it is noticeable that the excitations KK states of vector fields A_μ and scalar fields A_5 are the adjoint representations of gauge groups, while the complex weak doublet $H^{(n)}$ are the fundamental representations of $SU(2)$.

It is remarkable that, in the extra dimension scenarios we consider here, KK excitations always drive the

4D gauge couplings of $SU(3)$ and $SU(2)$ groups to their weak coupling limits. This is just the typical feature of non-Abelian gauge theory, i.e. the asymptotical freedom. Even for the case that all fermions in the SM (the universal case) are assumed to live in the bulk, the couplings of $SU(3) \times SU(2)$ are driven to their weak coupling limit.

To simplify our analysis, below we will omit the contributions of the $U(1)$ group, due to its small effect to the problem we consider here.

The Yukawa coupling terms of quarks in the SM are assumed to be the contact terms and have the form

$$L_{QUH} = Y_t \bar{Q}_L H U_R + h.c., \quad (14)$$

here $\bar{Q}_L = (\bar{u}_L, \bar{d}_L)$, $H^T = (H^0, H^-)$, and Y_F is the Yukawa couplings.

Due to its large effects, the Yukawa coupling of top quarks should be considered. The RGE of it can be written as $d\alpha_h/dt = b_h\alpha_h/(2\pi)$, and up to one-loop level b_h is defined as

$$b_h = -6C_c(1 + 2N_{KK})\alpha_3 - 3C_w(1 + 2N_{KK})\alpha_2 \\ + [N_c + \frac{3}{2}(1 + 2N_{KK})]\alpha_h, \quad (15)$$

where $\alpha_h = Y_t^2/(4\pi)$, and N_c is the number of color. The 2 before N_{KK} is due to the different normalization of zero modes and KK excitations. C_c and C_w are quadratic Casimir operators of $SU(3)$ and $SU(2)$ groups for fundamental representations, respectively.

The RGE of self-interaction coupling of Higgs zero mode fields λ can be expressed as $d\alpha_\lambda/dt = b_\lambda\alpha_\lambda/(2\pi)$, and up to one-loop, b_λ is defined as

$$b_\lambda = 3(1 + N_{KK})\alpha_\lambda + [(2C_w + 4)(1 + N_{KK}) \\ + (4C_w^2 + (2D_B + 8)c')\frac{\alpha_2}{\alpha_\lambda}(1 + N_{KK})] \alpha_2 \\ - 2N_c(2\frac{\alpha_h}{\alpha_\lambda} - 1)\alpha_h, \quad (16)$$

where $D_B = 5$, $\alpha_\lambda = \lambda/(4\pi)$, and $c' = 1 + N_w - 2/N_w + 1/N_w^2$. D_B is to count the degree of freedom of the gauge vector bosons. For $SU(2)$, $N_w = 2$. The first two terms in Eqn. (16) tend to increase α_l , and the last term tends to decrease it. While KK excitations of bosons always tend to drive α_λ to be large.

The one-loop RGE of Higgs mass term is expressed in the below form

$$\frac{dm_H^2}{dt} = \frac{1}{2\pi} \{ (\alpha_\lambda - 3C_w\alpha_2 + N_c\alpha_h)m_H^2 \\ + (\alpha_\lambda - 3C_w\alpha_2)N_{KK}m_H^2 \\ + ((\alpha_\lambda + (D_B - 2)C_w\alpha_2)M_{KK}^2) \}, \quad (17)$$

where $M_{KK}^2 = \sum_{i=1} m_i^2\theta(\mu - m_i)$, counting the contributions of KK excitations to the renormalization constant of m_H^2 .

In the UV, there are six free parameters in the effective theory needed to be specified, 1) α_3 and 2) α_2 , the fine structure constants of gauge group $SU(3) \times SU(2)$, 3) α_h , 4) α_l and 5) m_H^2 and 6) M_c . While in the low energy scale, there are only two free parameters needed to be specified 1) α_l , and 2) M_c , since α_3 , α_2 , α_h , and m_H^2 can be determined from experiments or those three free parameters. The $M_c = 1/R_c$ determines that where KK excitations should be counted. While triviality and stability conditions of Higgs potential in the SM constrain the value of $\alpha_\lambda(\mu_{IR})$ [14]. The Landau pole of α_λ can fix the value of Λ_{UV} .

We will concentrate on the analysis of the behavior of m_H^2 from the IR up to the UV. At the low energy scale (say $\mu_{IR} = M_z$, the m_H^2 is chosen to be negative as required by the SM. And its initial value at IR can be fixed by v (the VEV of H) and the free parameter $\alpha_\lambda(\mu_{IR})$.

Before the running scale cross the threshold of the first KK excitations, only the first term in Eqn. 17 contributes which tends to drive m_H^2 to be further negative. After only few KK excitations are counted, m_H^2 can change its sign from negative to positive, as shown in figure 1.

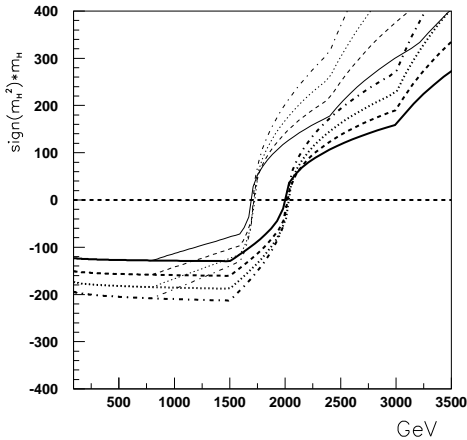


FIG. 1. The varying of the sign of m_H^2 and the value of m_H with the energy scale. The solid, dashed dot, and dash-dot lines represent $\lambda(M_z) = 1.0$, $\lambda(M_z) = 1.5$, $\lambda(M_z) = 2.0$ and $\lambda(M_z) = 2.5$, respectively. The group of wide(thin) lines corresponds to the case $M_c = 0.8$ TeV ($M_c = 1.5$ TeV).

As we know, in the SM, the sign of the mass term m_H^2 completely determines whether the electroweak symmetry is broken or unbroken. The change of m_H^2 from the negative to the positive value hints the restoring of the broken symmetry. With the same reasoning, as shown in our toy model, we can conclude it is possible to have the radiative SEWSB mechanism in the ED scenarios of the SM.

The underlying reason for the radiative SEWSB is that

the KK excitation of bosons are very heavy. With an appropriate value of $\alpha_\lambda(\mu_{IR})$, the process of the symmetry restoring can quickly happen after the running scale has crossed the threshold of the first KK excitation.

We also check the case in which only vector bosons live in the bulk, and find that with properly chosen M_c and λ , the radiative breaking mechanism exists too. For the universal case, we find that due to the large contribution from KK excitations of top KK excitations and the large top Yukawa coupling, it is relatively hard to find a appropriate point which can both invoke SEWB and preserve the validity of perturbation. However, in the universal case of the two Higgs doublets extension of the SM, with the help of $\tan\beta$ (the ratio of the VEV of the two Higgs doublets), it is still possible to have the radiative SEWSB mechanism induced by KK bosons.

In summary, we investigate the radiative electroweak symmetry breaking in the extra dimensions scenarios of the SM extension proposed by Antoniadis *et al.*, Dienes *et al.* and Pomarol *et al.*. Utilizing the decoupling theorem and the one-loop renormalization group equations of the parameters of Higgs potential, we find that heavy KK bosons can change the sign of the Higgs mass terms from positive to negative and therefore trigger the SEWSB. We conclude that the radiative mechanism can naturally exist in the ED scenarios, and new particle contents beyond the SM or SUSY are not necessary.

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- [1] P. Nath, hep-ph/0011177.
 - [2] Y. Kawamura, hep-ph/0012352.
 - [3] M. S. Chanowitz, Ann. Rev. Nucl. Part. Sci. **38**, 323 (1988).
 - [4] Hsin-Chia Cheng, hep-ph/0012263.
 - [5] Coleman, S. Weinberg, E. Phys. Rev. **D7**, 1888(1973).
 - [6] A. B. Lahanas and D. V. Nanopoulos, Phys. Rept. **145**, 1 (1987).
 - [7] I. Antoniadis, Phys. Lett. B **246**, 377 (1990); I. Antoniadis and K. Benakli, Phys. Lett. B **326**, 69 (1994) [hep-th/9310151].
 - [8] K. R. Dienes, E. Dudas and T. Gherghetta, Nucl. Phys. **B537**, 47 (1999), [hep-ph/9806292].
 - [9] A. Pomarol and M. Quiros, Phys. Lett. **B438**, 255 (1998) [hep-ph/9806263].
 - [10] M. Bando, T. Kugo, N. Maekawa and H. Nakano, Prog. Theor. Phys. **90**, 405 (1993) [hep-ph/9210229].
 - [11] J. A. Casas, V. Di Clemente and M. Quiros, Nucl. Phys. **B553**, 511 (1999) [hep-ph/9809275].
 - [12] H. Georgi, Annu. Rev. Nucl. Part. Sci. **43**, 209 (1993).
 - [13] T. Appelquist and J. Carazzone, Phys. Rev. **D11**, 2856 (1975).
 - [14] T. Hambye and K. Riessellmann, Phys. Rev. **D55**, 7255 (1997).